Difference Paradox

Consider two natural numbers n_1 and n_2, out of which one is twice as large as the other. We are not told whether n_1 is larger or n_2, we can state following two propositions:

PROPOSITION 1: The difference $n_1 - n_2$, if $n_1 > n_2$, is different from the difference $n_2 - n_1$, if $n_2 > n_1$.

PROPOSITION 2: The difference $n_1 - n_2$, if $n_1 > n_2$, is the same as the difference $n_2 - n_1$, if $n_2 > n_1$.

Moving on the proofs:

PROOF OF PROPOSITION 1: Let $n_1 > n_2$, then $n_1 = 2n_2$. Therefore, $n_1 - n_2 = 2n_2 - n_2 = n_2$. That's, the difference is n_2.

Let $n_2 > n_1$, then $n_1 = \frac{1}{2}n_2$. Therefore, $n_2 - n_1 = n_2 - \frac{1}{2}n_2 = \frac{1}{2}n_2$. That's, the difference is $\frac{1}{2}n_2$.

$\Rightarrow$ The proposition is true. $\square$

PROOF OF PROPOSITION 2: Let $n_1 > n_2$, then $n_1 - n_2 = n$, where n is a fixed natural number.

If $n_2 > n_1$, then $n_2 - n_1$ again equals to n.

Therefore, the proposition is true. $\square$.

If you re-read the propositions, you'll find that these are actually contradicting each others. But as both can't be true, which one should be treated as the correct one?

If you think that proposition 2 is true, then you need to reconsider and read again the propositions. Proposition 2 seems correct at first instance, but it isn't. *Take an example,* let $n_1 = 40$ and $n_2 = 20$, then $n_1 - n_2 = 20$. Again, let $n_1 = 10$ and $n_2 = 20$, then $n_2 - n_1 = 10$. Hence, the proposition 1 is true. $\square$

Reference: "A curious paradox", R. Smullyan

Original: https://gauravtiwari.org/glossary/difference-paradox/